

# Using a discount factor approach to solve for option constants in flexible investment.

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#### Abstract

Real option valuation rests upon boundary conditions where the arbitrary constants from general solution functions are determined by value matching and smooth pasting. The system of equations is linear in these constants but non-linear in the thresholds at which optimal investment takes place. This suggests that more progress can be made by starting with thresholds and solving for the constants as opposed to the other way around (which is standard in the literature).

For simple investment problems both approaches may work analytically but when the number of boundaries rises or decisions become cyclical (i.e. no final state), standard methods end with non-linear equation systems that require numerical solutions. We propose a more transparent matrix method that determines option constants across many decisions and thresholds, even if they are cyclical.

For decision networks of increasing scale, we represent value matching and smooth pasting as rows in a system of vectors. This requires the value of every option at two points (creation and use) but to offset the increase in constants we use a third set of conditions, namely discounting. We show how discounting produces the required smooth pasting conditions via option betas and how discounting can capture a decision network.

Keywords: Optimal stopping, smooth pasting, discount functions and investment graphs. C61, G31.

#### Highlights

- Provides a computational framework for multi level, multi stage decision making under uncertainty
- Places discounts from a decision network into a valuation matrix
- Given policy thresholds, provides explicit solutions for option values and decision costs
- Collates constants in suitable form to search numerically for policy thresholds that match specified costs.